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Rheological features of the 1971 Mount Etna lavas

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Rheological studies on the 1971 Mount Etna lavas indicate they underwent rapid transition from Newtonian to non-Newtonian fluids near their point of emission and that the non-Newtonian régime may be coincidental with high mechanical energy/low heat energy régime further from the boccas.

Darcy's equation quantifies the surface roughness of channels using the Chezy coefficient and is plotted against Reynolds number on a Stanton diagram. The relation is linear, and the critical value Re_c is not exceeded, proving wholly laminar flow.

The lava underwent a divergent, twin spiral motion involving two dimensional laminar flow. Convergent, twin spiral motion occurred only where lava passed through a constriction at a relatively high velocity.

Introduction

Quantitative observations on mobile lava were made during both the first and last phases of the recent eruption of Mt Etna. The eruption has been divided into four phases by Rittmann (1973, this volume) and others. The first phase lasted from 5 April to 6 May, and the last phase from 12 to 31 May.

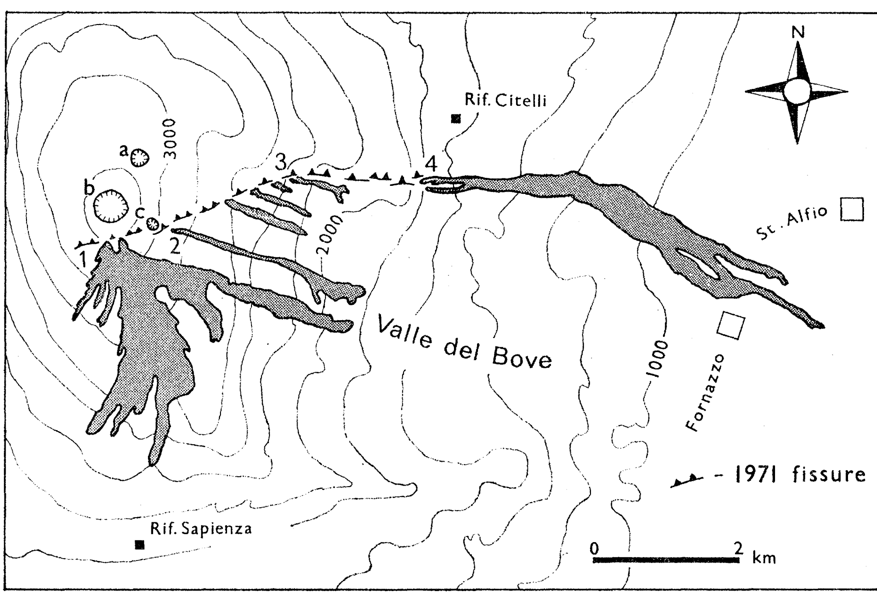

Figure 1. Sketch map of the 1971 Mt Etna lava flows. 1 to 4, sites of phases in the eruption; a, NE crater; b, central crater; c, 1971 explosion crater. Heights in metres.

The eastern extension of the 1971 fissure resulted in the first and fourth phases being separated by 5.5 km. Most attention in this paper is focused on the phase 4 flow, from near the Rifugio Citelli to Fornazzo, which was active for nearly one month. This flow proved most useful for consistent observations along its length: it erupted from a few, long lasting boccas at a fairly high rate of effusion, and followed the same course throughout its life, with a few minor lateral breakouts.

Measurements of flow velocity, dimensions of active lava channels, angle of slope and density of lava enabled the viscosity of the lava to be calculated, making it possible to apply fluid mechanics to analyse some of the flow phenomena observed.

Of the four phases comprising the 1971 eruption the first and fourth are most prominent, both in duration and the volume of lava produced (figure 1). The first phase released numerous subparallel flows from a moderate effusion into a broad, gently sloping valley and built up a compound lava flow consisting of many flow units (Booth 1972). During the fourth phase, lava flowed for most of the time from three boccas located in a small, steep-sided valley south of the Rifugio Citelli (figure 2). The three primary flows joined into one main flow about 200 m below the boccas; approximately 1 km downstream from the boccas a fourth, more vigorous flow, discharged into the main stream. The main lava stream proceeded to flow in a narrow valley east–southeast towards Fornazzo and into the Cava Grande gorge, where the furthest flow-front stopped 1.2 km below the village, a total of 7.3 km from its source. The higher rate of effusion and stronger topographic control resulted in a long, fairly narrow flow compared to phase one. The flow was most accessible in the upper reaches near the boccas, and in the lower reaches near Fornazzo.

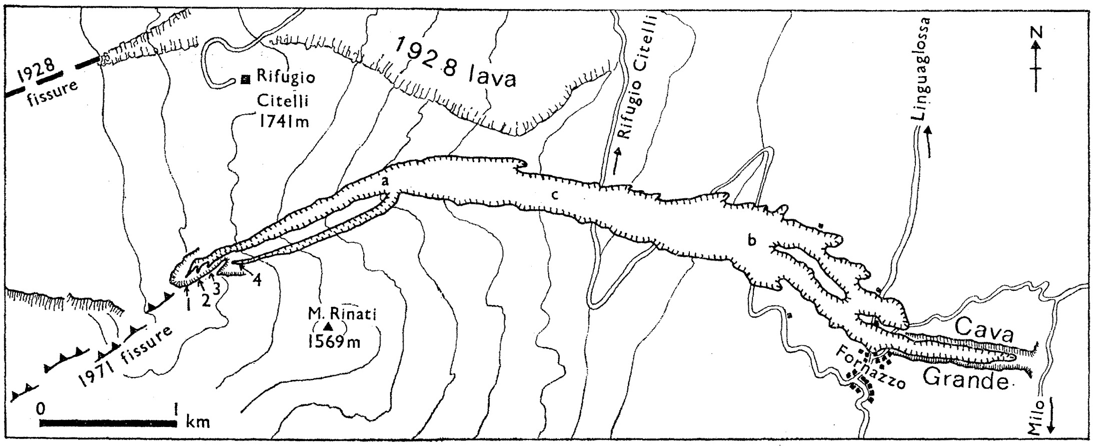

FIGURE 2. Sketch map of the 1971 Citelli phase 4 lava flows. 1 to 4, main Citelli boccas; a, zone of high heat energy/low mechanical energy; b, zone of high mechanical energy/low heat energy; c, transition zone between a and b.

The composition of the lava in all four phases was a tephritic phonolite, a trend towards mugearite beng detected at the end of phase 4 by Romano & Sturiale (1973, this volume). During the period of observation, 21 to 31 May, the lava composition was fairly constant and issued from the uppermost bocca (figure 2) at 2 to 3 m/s for most of the time. At the bocca the fluid lava appeared to 'swell up' (see also Greeley 1971) once under atmospheric pressure; this may be caused by a release of hydrostatic pressure outside the confines of the fissure, or to a hydraulic head effect as the lava appeared to have been flowing downslope before it reached the bocca, thus indicating a higher magma source, probably beneath the explosion crater at 2980 m (Booth & Walker 1973, this volume) 1000 m above the boccas to the west–southwest (figure 1).

The molten lava is a three-phase liquid composed of fluid lava, crystals and gas bubbles. The proportion of bubbles in a lava can markedly increase its viscosity (Einarsson 1949), but the phase 4 lavas were low in gas content, most gases being lost from the explosion crater, so that this effect can be considered small compared with the increase of viscosity due to temperature drop and crystallization.

One difficulty in making meaningful recordings of viscosity is the braided-stream nature of the flow as a whole, and most observations were made on one or two active flow units whose channels carried the bulk of the lava during the period 21 to 31 May. In the upper parts, the channels varied little in shape, apart from levee construction and increase in channel floor thickness, due to the confines of the narrow, forested valley down which the lava flowed.

Application of formulae and dimensional analysis

Most theoretical fluid mechanics applies to low viscosity, Newtonian fluids flowing in uniform pipes or channels down uniform slopes. Many viscosity measurements have been made on lavas in the field and laboratory in the past, but very few workers have attempted to apply fluid dynamics to flowing lava. Nichols (1939) showed that Hawaiian lavas underwent laminar flow, where laminar flow can be defined as a flow within which any unit of liquid contains filaments that are all parallel. Laminar flows obey Newton's law of fluid motion, which can be expressed as

$$t = -\mu(\mathrm{d}V/\mathrm{d}y), \tag{1}$$

where μ is the coefficient of viscosity, t the shearing force per unit area, and $\mathrm{d}V/\mathrm{d}y$ the velocity gradient or shear rate.

Before Nichols's findings (1939) and subsequent work by Wentworth, Carson & Finch (1945), and Wentworth (1954), it was considered that fluid lava displayed turbulent flow. This is essentially a flow with random eddies whose velocities are faster than the total velocity of the flow.

Fluids are divisible into newtonian and non-newtonian types, depending on whether they obey Newton's relation (1) above. Non-Newtonian fluids are high-viscosity liquids which do not flow unless the shearing stress (t) exceeds a certain value peculiar to that fluid, they exhibit only laminar flow such as plastic flow (deformation), whose behaviour is particularly applicable to geological systems. On a Newtonian model the viscosity is easily defined, and analysis shows that at least the hottest and most fluid lava near the bocca is a Newtonian fluid. During its course to the flow front the lava must develop non-Newtonian properties; the necessary shear stress (t) required to maintain flow being produced by the mass of molten material flowing downslope from the boccas towards the flow front.

As shown on figure 2, there is an exchange from high heat energy/low mechanical energy to low heat energy/high mechanical energy between the bocca and the flow front. Whether the non-Newtonian régime is coincidental with the high mechanical energy régime is conjectural and both exchanges are expected to be extremely gradual.

The viscosity of the lava was calculated at many points down the length of the flows. Viscosity was calculated using a formula, which is derived from Newton's relationship:

$$\mu = g\rho h^2 (\sin \alpha)/3V, \tag{2}$$

where g is the acceleration due to gravity, ρ the density of lava, h the depth of flow, α the angle of slope, and V velocity of lava.

The factor $3V$ is used when the flow is contained in wide channels (i.e. at the distal end of the flow), whereas for narrow channels such as those near the boccas, $4V$ is used (Minakami 1951).

V, h and α were measured in the field: ρ was measured by determining the mass and volume of five samples pulled from the molten lava and quenched in water. This method may give a slightly higher value for ρ, but the error is not considered significant compared with the magnitude of the viscosities. A mean value of 2.6 g/cm^3 was taken for ρ.

Figure 3 indicates the range in viscosities obtained for the first and fourth phases. For comparison asphalt, at room temperature, has a viscosity of 10^6 Pa s (10^7 P). It can just be deformed by hand, but not broken; if knocked it shatters conchoidally (Einarsson 1949). The lava at the boccas was very fluid, yet dense stones thrown onto the moving surface did not sink and often rebounded off, it was flowing at 3 to 4 m/s with a viscosity of 10^2 to 10^3 Pa s (10^3 to 10^4 P).

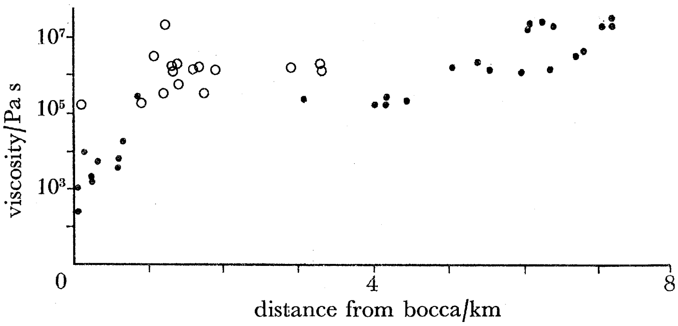

FIGURE 3. Plot of viscosity against distance from bocca for phase 1 (○) and phase 4 (●).

The depth of flow (h), proved more difficult to measure in the field and the values can be no more than reasonable estimates. Evidence from evacuated channels, breakouts and the smaller phase 1 flows indicates that width (a) of channels is comparable to, or slightly greater than, depth (h). This applies only to the upper parts of flows, and excludes exceptional circumstances such as ponding and severe topographic restriction. Measurement at such places was avoided.

As all field measurements are subject to systematic errors, the validity of the viscosities must be to an order of magnitude only. Two sets of measurements were taken on the same flow, only a few metres apart, but under different flow conditions of channel shape and slope. There was no ponding, so the mass flow rate must have been similar. The two viscosities obtained were 8.7 and 6.3 kPa s (8.7 and 6.3×10^4 P). This may indicate the accuracy of the results.

TABLE 1. TEMPERATURES AND VISCOSITIES OF 1971 ETNA AND OTHER LAVAS

	temperature of lava surface/°C	viscosity Pa s	viscosity P
(i) Etna phase 1 lavas (Gauthier, personal communication)	1120–1125	10^4–10^5	10^5–10^6
	1100	10^5	10^6
(ii) Etna lava: laboratory determination (Gauthier, personal communication)	1100–1150	10^3	10^4
(iii) Etna Citelli boccas (Tanguy, personal communication)	1100–1130	10^2–10^3	10^3–10^4
(iv) Etna lava (le Guern, personal communication)	1040	10^2–10^3	10^3–10^4
(v) Hawaii: experimental data on olivine basalt (Nichols 1939)	1200	3.18×10^2	3.18×10^3
	1150	3.79×10^3	3.79×10^4
(vi) Hawaii: field determinations (McDonald 1963)	900–1000	10^2–10^3	10^3–10^4

The range of recorded viscosity for the Etna flows is 10^3 to 10^7 Pa s (10^4 to 10^8 P), the lower values being most meaningful. These were all measured near the bocca and are within the previously determined ranges; Macdonald (1963) gives 10^2 to 10^3 Pa s (10^3 to 10^4 P) for Hawaiian tholeiitic basalt and Einarsson (1949) 10^4 Pa s (10^5 P) for Icelandic andesitic lava. One viscosity of the order of 10^2 Pa s (10^3 P) was determined early in May but this value has been derived from the analysis of cine film of the lava.

Tanguy (personal communication) also recorded 10^2 Pa s (10^3 P) in the early days of the Citelli boccas. As expected, a linear relation between viscosity and distance from source is found (figure 3), due to the effects of gradual cooling, crystallization and reduction in slope angle. Some temperature measurements for the phase 4 lava are given in table 1 together with comparable ones from earlier works.

The transition between laminar and turbulent flow is best expressed by the Reynolds number (*Re*) of the fluid. Values of *Re* up to approximately 2000 are indicative of laminar flow, above which the régime of turbulent flow is reached, but the critical value of *Re* is ill defined. *Re* is computed:

$$Re = DV\rho/\mu, \tag{3}$$

where D is the diameter of pipe, V the velocity of flow, ρ the density of fluid, and μ the viscosity of fluid; *Re* is dimensionless.

Most fluid mechanics involves ideal cases of uniformly circular pipes or rectangular open channels. The cross-sectional form of a lava flow is often somewhere between these two extremes and considering this, the standard fluid mechanics formulae should be applicable to lavas, with D_e substituted by $4R_h$, where D_e = another characteristic linear dimension of the solid boundary and $D_e \equiv D$, where R_h is the hydraulic radius of the channel

$$D_e = 4\left(\frac{\text{cross-sectional area}}{\text{wetted perimeter}}\right).$$

Many of the channels where observations were made had approximately equidimensional cross-sections, so if $a = h$, where h = depth and a = width and

$$D_e = 4\,\frac{ah}{2(a+h)}, \tag{4}$$

then $D_e = h$; i.e. the pipe diameter D can be substituted by depth in certain cases. Otherwise $4R_h$ is used. Several calculations were made twice, once with h, and again with $4R_h$: the values of *Re* obtained were not significantly different, considering that the viscosity value used is correct only to an order of magnitude. Very few *Re* values for lavas have been computed in the past; one of approximately 36 for Hawaiian basalt is given by Einarsson (1949).

Re values for the Etna lavas are all very small and never approach the turbulent flow régime. To test the feasibility of a laminar flow model, *Re* was plotted against a friction factor (f), the Darcy friction factor, which is computed from the same parameters as the viscosity:

$$f = 8gsh/V^2, \tag{5}$$

where g is the acceleration due to gravity, s the slope ($\sin\alpha$), h the channel depth, and V velocity of lava flow.

A plot of lg *Re* against f is known as a Stanton diagram and is a well tested way of defining the fields of laminar and turbulent flow. The slope of the graph is important in defining laminar flow where $Re < 10^3$. Figure 4 shows a Stanton diagram for the 1971 phase 4 lava flows from which the flows appear to be wholly laminar.

Very little is known quantitatively about the nature of the channel floor over which the lava flows. From field observations it appears to be rough, the channel dimensions are not uniform and can vary from a closed tunnel to a wide open channel. The surface roughness of channels can be quantified, using a formula derived from Darcy's equation (5):

$$f = 8gsh/V^2, \tag{5}$$

$$V = \sqrt{(2gR_\mathrm{h}s/f)} \quad \text{or} \quad V = C\sqrt{(R_\mathrm{h}s)}, \tag{6}$$

R_h is the mean hydraulic radius, s the slope, C the Chezy coefficient $= \sqrt{(2g/f)}$.

Analysis of C and R_h by Manning, gave

$$C = R_\mathrm{h}^{\frac{1}{6}}/n, \tag{7}$$

where n is the Manning coefficient (a coefficient of surface roughness).

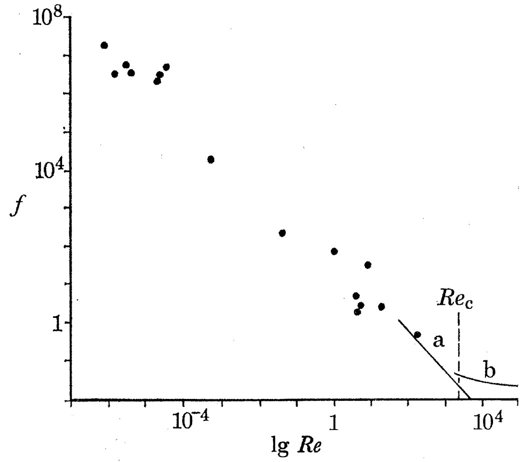

FIGURE 4. Plot of Darcy friction factor (f) against log Reynolds number (*Re*); a, laminar flow régime; b, turbulent flow régime; Re_c, critical value of lg *Re*.

These relations all apply to ideal cases such as uniform channels, therefore computed values for n are only approximate. Table 2 gives values of n for the 1971 Etna lavas and some other surfaces for comparison.

TABLE 2. MANNING COEFFICIENT VALUES

surface	n ($\mathrm{m}^{-\frac{1}{3}}$ s)
concrete	0.011–0.017
brick	0.012–0.020
earth	0.020–0.030
gravel	0.022–0.035
rock	0.030–0.080
(i) Etna lava	0.07
(ii) Etna lava	2.20

(i) Main channel below Citrelli bocca 3.
(ii) Main channel 4.3 km below boccas.

APPLICATION OF RESULTS TO FIELD OBSERVATIONS OF FLOW

A rotational movement was observed in the fast flowing (1 to 3 m/s) lavas near the boccas of both 1 and 4 phases. Such rotations have been previously recorded by Krauskopf (1948), Einarsson (1949) and Greeley (1971), but only in special conditions of flow and have only been observed to take place during the 1971 Etna eruption where the lava flows were contained by levees or deep channels. In the Etna lava such motion was commonplace while a molten surface existed and probably operated under the scoriacious surface. A particular flow in its narrow channel showed a slow, rotational movement that could be mistaken for turbulence. This rotation is related to the surface velocity profile. Considering particles of lava within the coherent molten flow, analysis of the above-mentioned movements in the field and on cine film indicates that particles rise in the centre of the lava stream causing a marked upwelling of the middle third of the flow. This middle portion is defined by a band of darker, cooler surface scales which are able to cool because they remain in the central zone, the maximum velocity zone, for a comparatively long time before they are dragged quickly to the side of the flow, down a velocity gradient, by wall friction. At the centre of the middle zone, forming a line of symmetry in the flow, is a bright glowing seam; this is probably an effect of upwelling hotter material coming to the surface of the flow. This is consistent with the expected cross-sectional velocity and heat profiles through a flow.

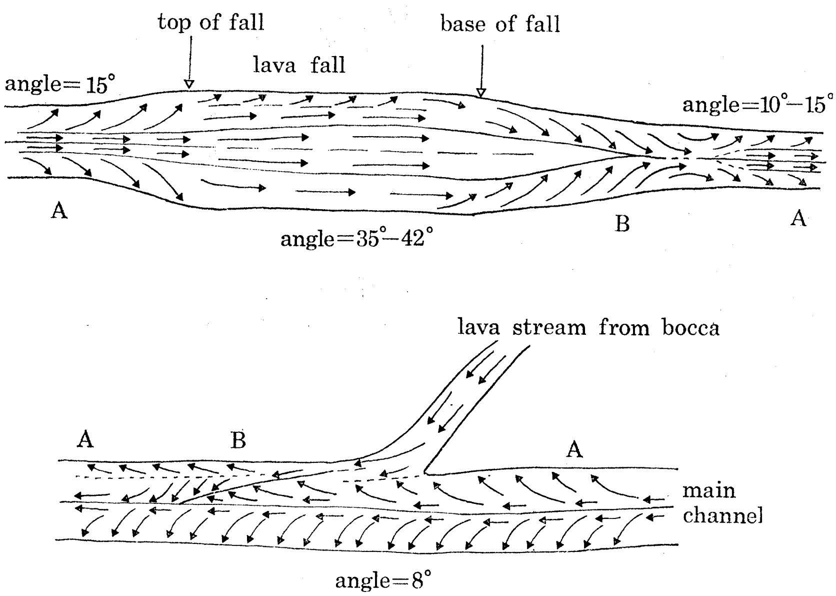

Figure 5. Lava fall below bocca 4, Citelli flow. Upper diagram; A, divergent flow régime; B, convergent flow régime. The lines in the central part of the diagram indicate 'seams', i.e. flow-formed boundaries on the lava surface. The height between the top and base of the fall is approximately 20 m and the maximum slope 42° Lower diagram: (confluence of two lava flows below bocca 2 (30 May 1971).

It must follow that when material moves to the side in a flow channel it must, for the most part, undergo a divergent twin spiral motion and recirculate. If this were not so, then the lava channel would rapidly narrow downslope from the boccas due to material sticking to the channel walls. Yet during periods of continued observation the channels in the region of the boccas did not become systematically narrower, either downslope or with time. However, over the period during which the flows were active, the floor of the lava channels increased in thickness due to material undergoing this rotational movement adhering to their surface, but compared to the volume of material passing through these upper channels this can be considered small.

This divergent, twin spiral motion operates each side of the centre seam on the lava surface and probably involves a two-dimensional laminar flow. Any possibility of a turbulent flow within the viscosity range calculated must be ruled out. This spiral motion is thought to be driven by (1) lava from behind moving faster along the centre third of the flow, (2) the frictional drag of the channel side, and (3) a possible central convection effect from the heat energy stored in the lava.

Convergent, twin spiral motion was only observed to take place where the lava passed through a constriction at high velocity and at the confluence of two rapidly moving lava flows (figure 5). In this case the lava at the sides was pushed up against the channel walls as it approached the narrow channel and was folded over on itself as it passed through the constriction.

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